Impact of motion along the field direction on geometric-phase-induced false electric dipole moment signals

H. Yan, B. Plaster

Department of Physics and Astronomy, University of Kentucky, Lexington, KY, 40506 USA

Abstract

Geometric-phase-induced false electric dipole moment (EDM) signals, resulting from interference between magnetic field gradients and particle motion in electric fields, have been studied extensively in the literature, especially for neutron EDM experiments utilizing stored ultracold neutrons and comagnetometer atoms. Previous studies have considered particle motion in the transverse plane perpendicular to the direction of the applied electric and magnetic fields. We show, via Monte Carlo studies, that motion along the field direction can impact the magnitude of this false EDM signal if the wall surfaces are rough such that the wall collisions can be modeled as diffuse, with the results dependent on the size of the storage cell's dimension along the field direction.

Keywords: neutron electric dipole moment, geometric phase false EDM, wall collisions

1. Introduction

If a non-zero neutron electric dipole moment (EDM) is observed, such a discovery would provide evidence for parity-violation and time-reversal-symmetry-violation beyond that of the standard model [1]. Neutron EDM searches are usually based on a Nuclear Magnetic Resonance (NMR) technique, in which the Larmor precession frequencies are compared for parallel and anti-parallel (weak) magnetic and (strong) electric field configurations. A value for, or a limit on, the EDM is then extracted from the frequency difference for these two field configurations. All recent experiments have been

Preprint submitted to Nucl. Instrum. Methods Phys. Res. A *January 19, 2013*

performed with ultracold neutrons (UCN), neutrons with speeds less than $\lesssim 7$ m/s, or energies $\lesssim 350$ neV [2]. Their low speeds permit storage in cells for long periods of time (in principle, up to the β-decay lifetime), and reduce systematic errors related to the neutron velocity. Of crucial importance to all EDM searches is monitoring of the magnetic field. This is typically accomplished using in-situ co-magnetometer and/or external magnetometer atoms, such as ^{199}Hg [3], ^{3}He [4, 5], and ^{133}Cs [6].

A systematic error that has been discussed extensively in the literature recently [7, 8, 9, 10] is the so-called geometric phase effect, resulting from interference between magnetic field gradients and the $(\vec{E} \times \vec{v})/c^2$ motional magnetic field in the particle rest frame. The result is a frequency shift $\delta\omega$ proportional to the electric field E, for both the UCN and magnetometer atoms, which is dependent on the particles' (geometric) trajectories. Because the frequency shift is proportional to E, the effect could be interpreted as a false EDM signal. A general description of this effect based on the density matrix formalism was developed [8, 9, 10], in which the frequency shift was shown to be proportional to the Fourier transformation of the velocity autocorrelation function for a constant gradient $\partial B_{0z}/\partial z$ (assuming fields along the z-axis). The frequency shift $\delta\omega$ was shown to be of the form [8, 9, 10]

$$\delta\omega = \frac{\gamma^2}{4}\frac{\partial B_{0z}}{\partial z}\frac{E}{c^2}\int_0^t d\tau R(\tau)\cos\omega_0\tau \tag{1}$$

where the autocorrelation function is

$$\begin{aligned} R(\tau) &= \langle y(t)v_y(t-\tau) + x(t)v_x(t-\tau) \\ &\quad - y(t-\tau)v_y(t) - x(t-\tau)v_x(t)\rangle \\ &= 2\int_0^\tau \langle v_y(t)v_y(t-\tau) + v_x(t)v_x(t-\tau)\rangle dt. \end{aligned} \tag{2}$$

Here, γ is the gyromagnetic ratio, and $\omega_0 = \gamma B_{0z}$ is the nominal precession frequency for a magnetic field B_{0z}.

2. Motion Along the Field Direction

As was pointed out in [8], both wall and gas collisions can lead to a suppression of the autocorrelation function, and thus of the frequency shift. However, upon inspection of Eq. (2), it can be seen that if we consider three-dimensional motion, the velocity component along the z-axis, v_z, may impact

the autocorrelation function, even though v_z itself does not contribute to the $(\vec{E} \times \vec{v})/c^2$ field. The reasoning is as follows. Suppose a particle undergoes a collision with a "z-wall". If the wall collision is specular, the velocity components before (v) and after (v') the collision will transform according to

$$v'_x = v_x, \qquad v'_y = v_y, \qquad v'_z = -v_z, \tag{3}$$

and thus specular wall collisions will have no impact on the autocorrelation function $R(\tau)$. However, for collisions with rough wall surfaces, such that the reflection angles can be modeled as Lambertian diffuse, the v_x and v_y velocity components before and after the collision will not, in general, be correlated

$$v'_x \neq v_x, \qquad v'_y \neq v_y, \qquad v'_z \neq -v_z. \tag{4}$$

This suggests that an understanding of the geometric-phase false EDM effect in future experiments will require detailed studies of the autocorrelation functions of the stored UCN and co-magnetometer atoms, and that these functions will depend on the type of wall collisions the stored particles undergo, and on the size of the storage volume along the field direction (thus affecting the wall collision rate). We have employed the autocorrelation function approach of [8, 9, 10] in Monte Carlo studies of the impact of particle motion along the field direction (i.e., velocities along the z-axis) on the magnitude of the geometric-phase-induced false EDM signal. In the next section, we show several illustrative results from our Monte Carlo studies of this effect.

3. Results from Monte Carlo Studies

Monte Carlo codes were developed to study the impact of motion along the field direction on the autocorrelation function. Collisions with the wall surfaces were modeled as either specular or Lambertian diffuse (i.e., reflection angles sampled from a $f(\theta)d\Omega \propto \cos\theta d\Omega$ angular distribution). For UCN, we sampled a v^2dv velocity distribution up to some cut-off velocity, and for co-magnetometer atoms, we sampled a Maxwell-Boltzmann distribution. Below we discuss several example results from our Monte Carlo studies.

First, we demonstrate in Fig. 1 that the autocorrelation function is, indeed, insensitive to the three-dimensional size of the storage volume for purely specular wall collisions. This figure shows plots of $R(\tau)$ for UCN stored in a 10 cm $\times$ 10 cm two-dimensional square geometry, for different z-dimensions.

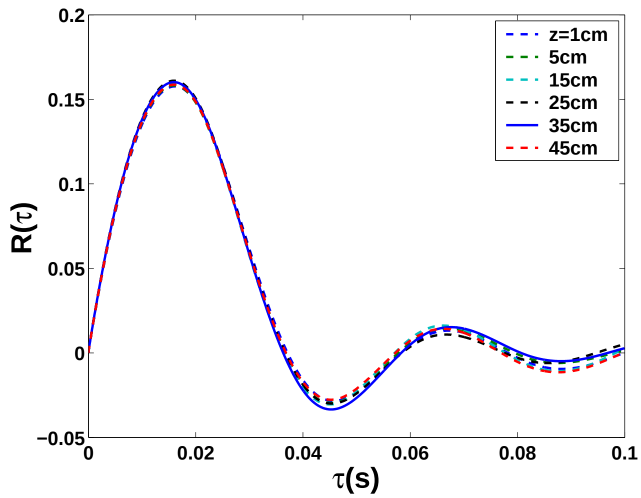

Figure 1: (Color online) Results from simulations of the autocorrelation function $R(\tau)$ for UCN undergoing specular wall collisions in a 10 cm $\times$ 10 cm two-dimensional square geometry, for the different indicated z-dimensions.

Indeed, there is little difference between these results. Thus, for specular wall collisions, simulations in two-dimensions are sufficient.

Second, Fig. 2 shows results from simulations for wall collisions modeled as purely Lambertian diffuse. Here, it can be seen that as the z-dimension increases, the results approach the limiting two-dimensional result in which the collision rate with the z-walls becomes small relative to the collision rate with the walls in the transverse x-y plane. In the opposite limit, as the z-dimension becomes small relative to the transverse x-y dimensions, the collision rate with the z-walls increases, and the autocorrelation function becomes highly suppressed.

Third, as a final example, we considered the false EDM of the ^{199}Hg co-magnetometer atoms stored in the cylindrical geometry of the neutron EDM experiment reporting the current benchmark upper limit of $< 2.9 \times 10^{-26}$ e-cm (90% C.L.) [3]. As reported in [7], for a gradient of $\partial B_{0z}/\partial z = 1$ nT/m at a field of $B_{0z} = 1$ μT, or a fractional gradient of $(\partial B_{0z}/\partial z)/B_{0z} = 10^{-5}$ cm^{-1}, the false EDM signal of the Hg atoms was (using their notation) $d_{af\mathrm{Hg}} = 1.3 \times 10^{-26}$ e-cm.

Our simulations assumed the ^{199}Hg atoms were at a temperature of 300 K (with no buffer gas, thus undergoing wall collisions only), the radius of the

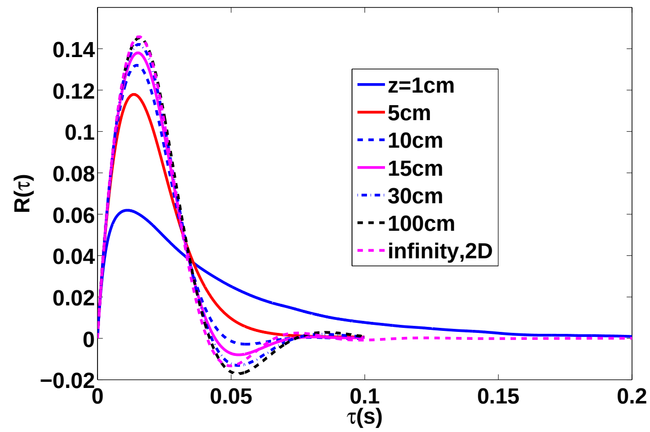

Figure 2: (Color online) Results from simulations of the autocorrelation function $R(\tau)$ for UCN undergoing Lambertian diffuse wall collisions in a 10 cm $\times$ 10 cm two-dimensional square geometry, for the different indicated z-dimensions.

cylindrical geometry was 0.25 m (as reported in [7]), and the nominal three-dimensional size (height) of the cylidrical geometry was 0.12 m (as reported in [11]). Some results of our simulations are shown in Fig. 3, where we have plotted the false EDM signal of the ^{199}Hg atoms, $d_{af\mathrm{Hg}}$, versus the magnetic field B_{0z}. As a consistency check, we note the good agreement at $B_{0z} = 1\ \mu$T between the result obtained in our simulations, and the value of 1.3×10^{-26} e-cm reported in [7]. The electric field was assumed to be $E = 10$ kV/cm [3].

At the operating magnetic field of the previous experiment [$B_{0z} = 1\ \mu$T (= 10 mG)], the differences between the two-dimensional case and the three-dimensional case are much smaller than the extracted EDM limit. However, as the value of the magnetic field increases (i.e., as the precession frequency becomes large relative to any angular velocity of the stored particles), the differences between the two-dimensional case and the three-dimensional case become apparent, with the magnitude of the false EDM showing a significant dependence on the exact size of the dimension along the z-axis (as demonstrated for the two other, arbitrarily chosen, different z-dimension sizes).

4. Discussion

In summary, our Monte Carlo studies suggest that in neutron EDM experiments with stored UCN and co-magnetometer atoms, if the wall surfaces are

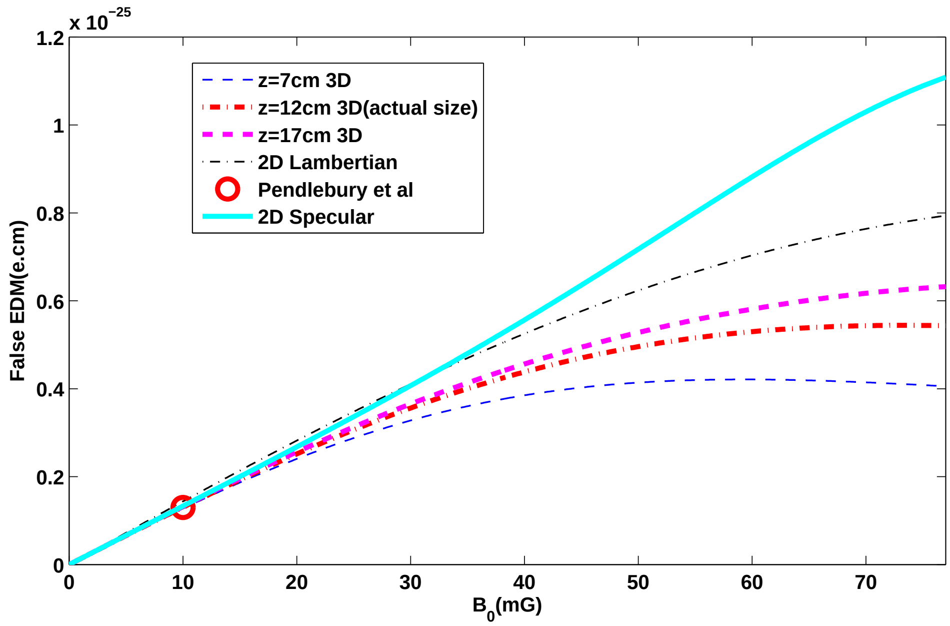

Figure 3: (Color online) Results from simulations of the false EDM of ^{199}Hg co-magnetometer atoms stored in a cylindrical geometry with a radius of 0.25 m, and for various three-dimensional heights. The fractional field gradient is 10^{-5} cm^{-1}. The red circle indicates the value for $d_{af\mathrm{Hg}}$ reported in [7].

rough such that the collisions are modeled as Lambertian diffuse, the magnitude of the geometric-phase-induced false EDM signal will be dependent on the size of the storage cell's dimension along the field direction. Indeed, as expected intuitively, the magnitude of the false EDM can be suppressed by utilizing a geometry with a relatively small dimension along the field direction. For planning of future neutron EDM experiments, our studies suggest that it will be important to understand all possible physical processes which may impact the autocorrelation function, including quantifying the degree to which the wall collisions can be modeled as either specular or diffuse, as presented in this work. Indeed, a complete understanding of the geometric-phase-induced false EDM signal in future experiments will require experimental measurements of the autocorrelation function, coupled to three-dimensional simulations.

Acknowledgments

We thank B. Filippone and R. Golub for valuable discussions. This work was supported in part by the U.S. Department of Energy under award number DE-FG02-08ER41557, and by the University of Kentucky.

References

[1] M. Pospelov and A. Ritz, Ann. Phys. **318**, 119 (2005).

[2] R. Golub, D. J. Richardson, and S. K. Lamoreaux, *Ultra-Cold Neutrons* (Hilger, Bristol, 1991).

[3] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006).

[4] R. Golub and S. K. Lamoreaux, Phys. Rept. **237**, 1 (1994).

[5] S. K. Lamoreaux and R. Golub, J. Phys. G Nucl. Part. Phys. **36**, 104002 (2009).

[6] I. Altarev *et al.*, Nucl. Instrum. Methods Phys. Res. A **611**, 133 (2009).

[7] J. M. Pendlebury *et al.*, Phys. Rev. A **70**, 032102 (2004).

[8] S. K. Lamoreaux and R. Golub, Phys. Rev. A **71**, 032104 (2005).

[9] A. L. Barabanov, R. Golub, and S. K. Lamoreaux, Phys. Rev. A **74**, 052115 (2006).

[10] R. Golub and C. M. Swank, `arXiv:0810.5378`.

[11] P. G. Harris and J. M. Pendlebury, Phys. Rev. A **73**, 014101 (2006).